CS173: Graphs and 2-Way Bounding

Gul Agha

Presented by Karl Palmskog

Based on slides by Derek Hoiem

September 29, 2016

CS173 B-lecture Announcements

- **No** examlet this week
- A Moodle Quiz is due **Friday evening**

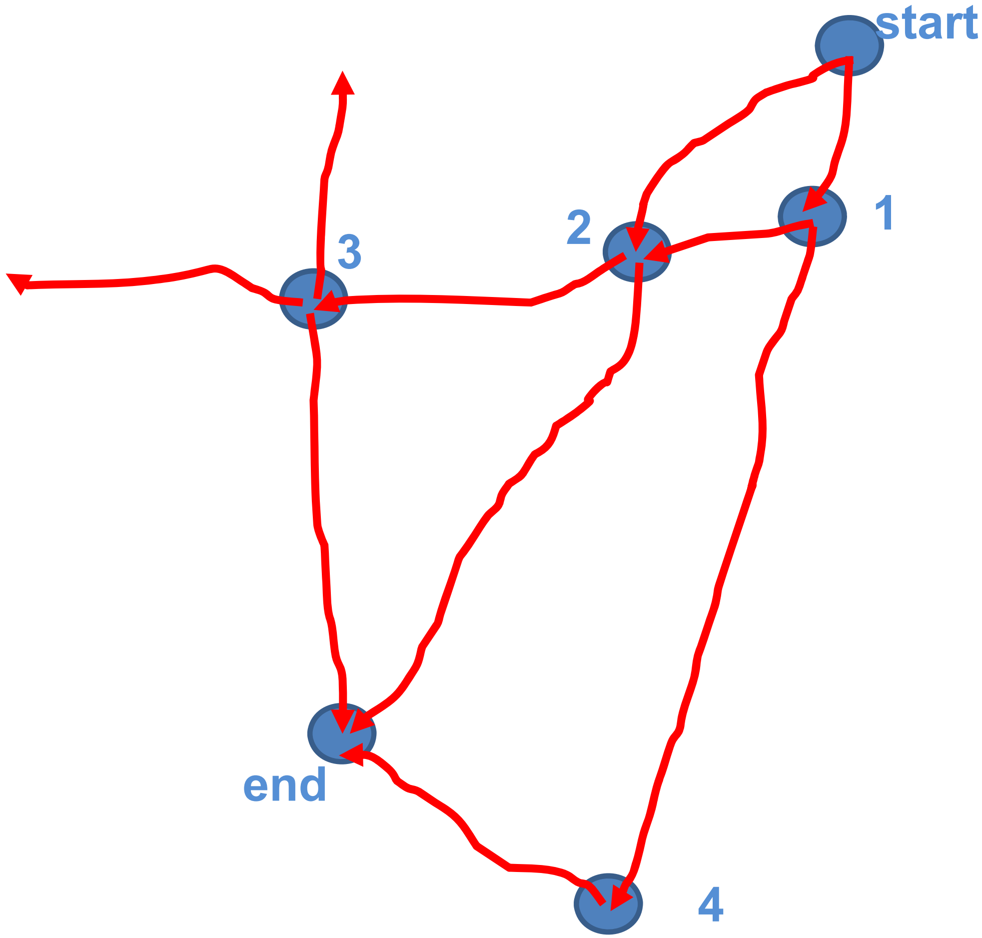
Last Lecture: Graphs

- How to represent graphs?
- What are the properties of a graph?
  - Degrees, special types
- When are two graphs isomorphic, having the same structure?

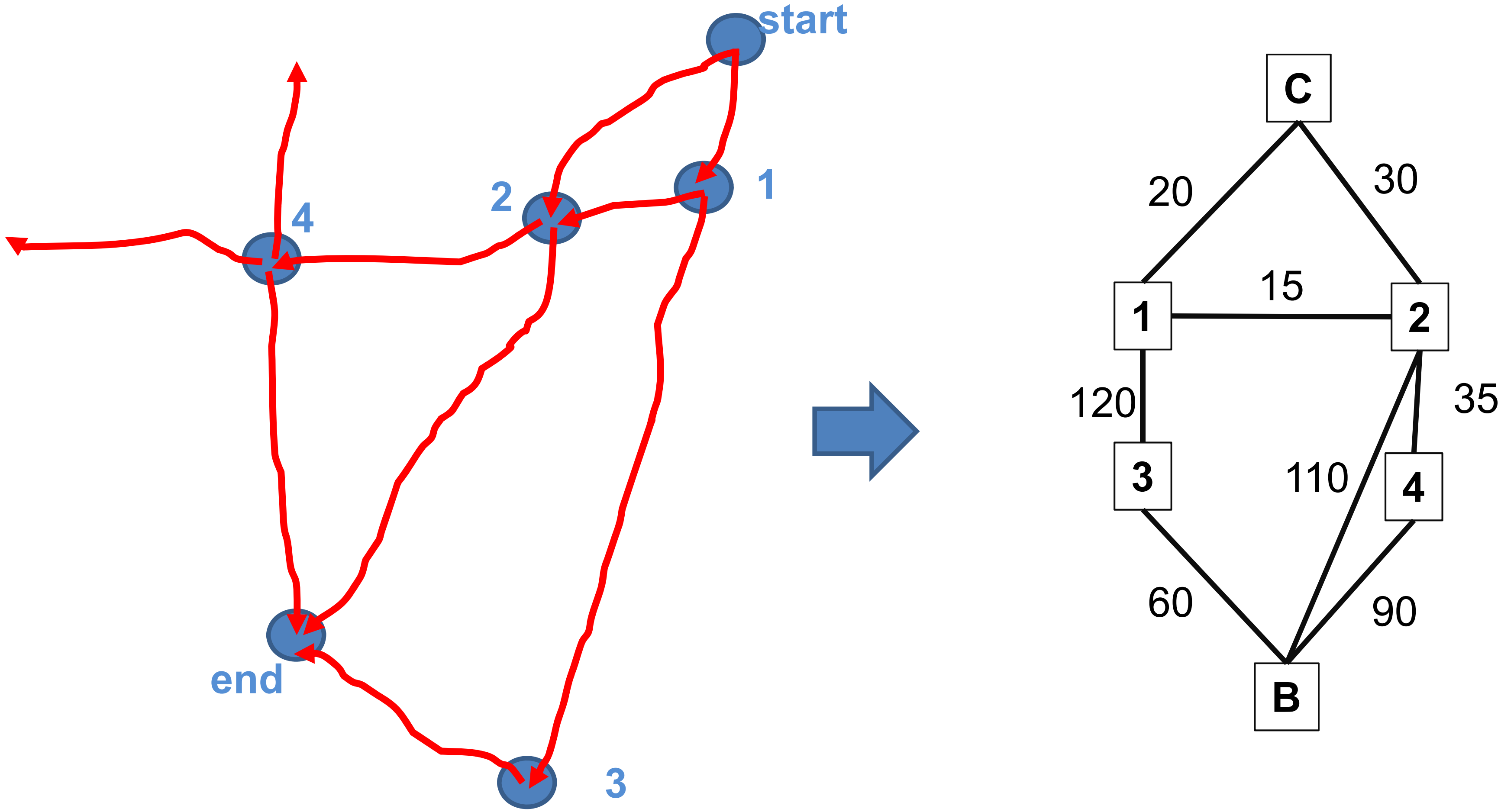
Fastest path from Chicago to Bloomington?

Fastest path from Chicago to Bloomington?

Fastest path from Chicago to Bloomington?

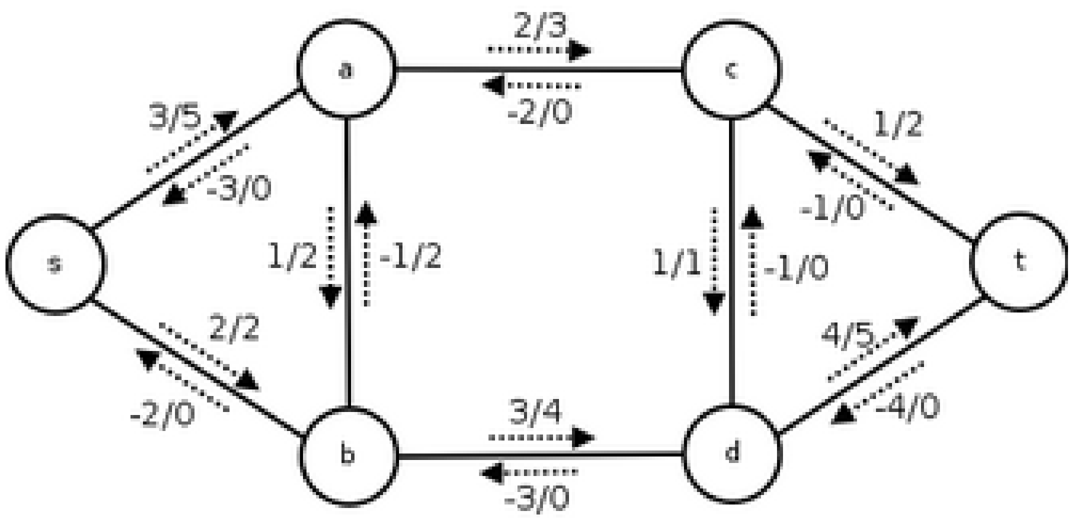


Fastest path from Chicago to Bloomington?



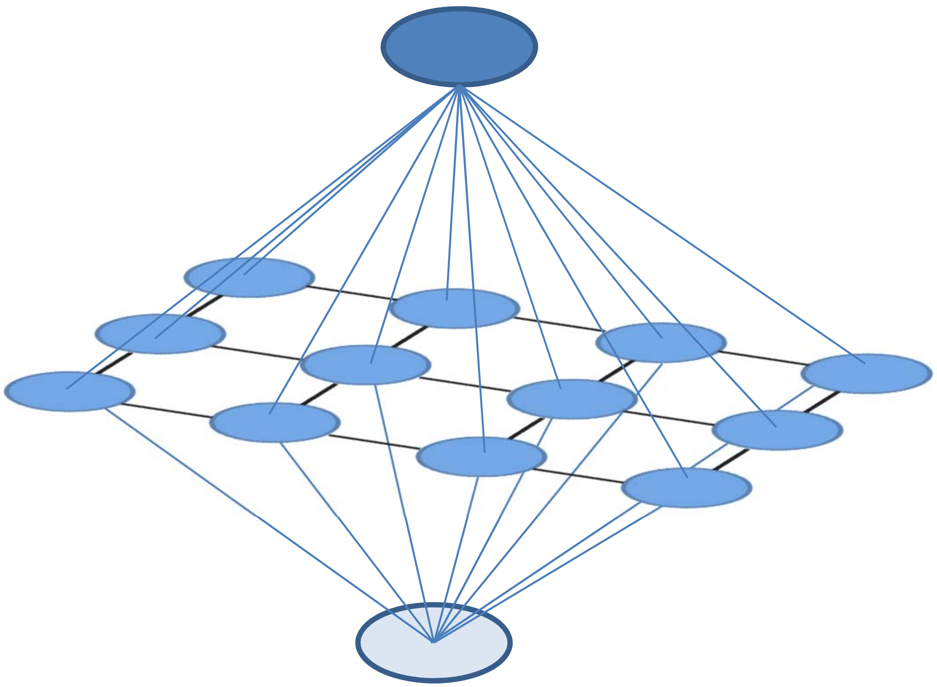
Other applications of graphs

- Modeling the flow of a network
  - Traffic, water in pipes, bandwidth in computer networks, etc.

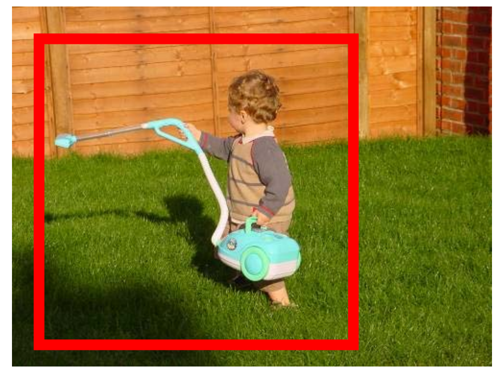


Other applications of graphs

- Modeling probabilistic dependencies

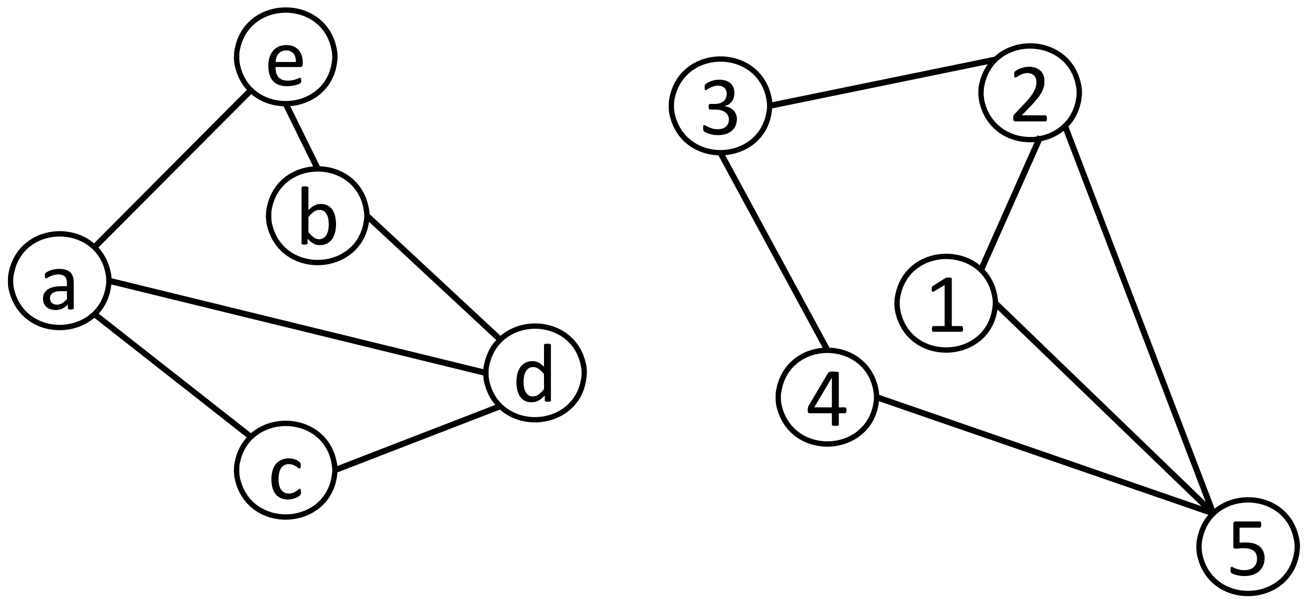


Graphical model of dependencies between pixels for object segmentation



Segmented object with “GrabCut” by Rother et al. 2004

Isomorphism example



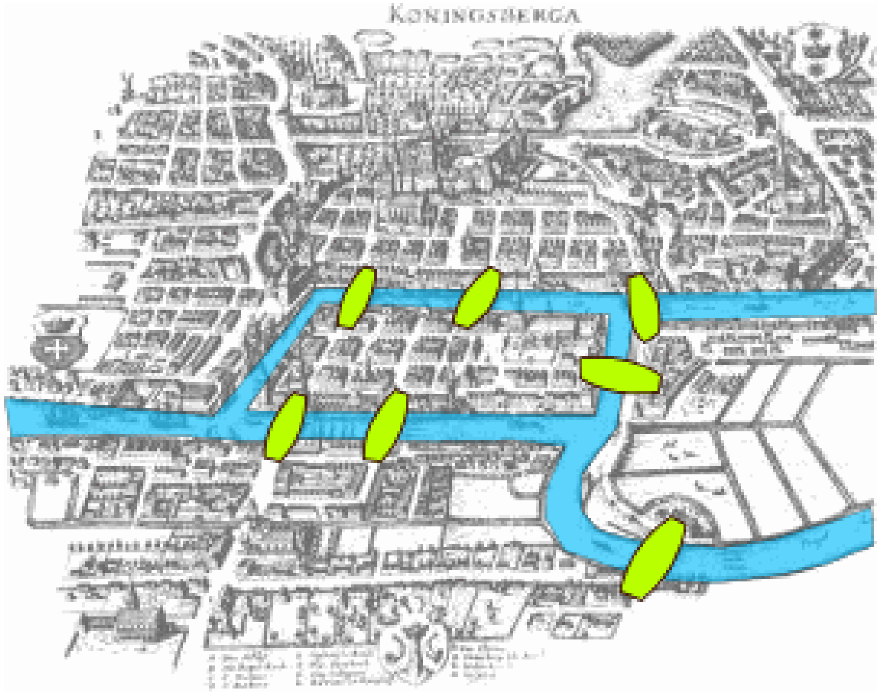
overhead

Today's lecture

- How do we characterize connectivity and walks in graphs?
- What is graph coloring and what is it good for?
- What are bipartite graphs and why are they important?
- Methods and applications for 2-way bounding

Bridges of Konigsberg

Possible to cross all bridges exactly once and end up where you started?



First theorem of graph theory!

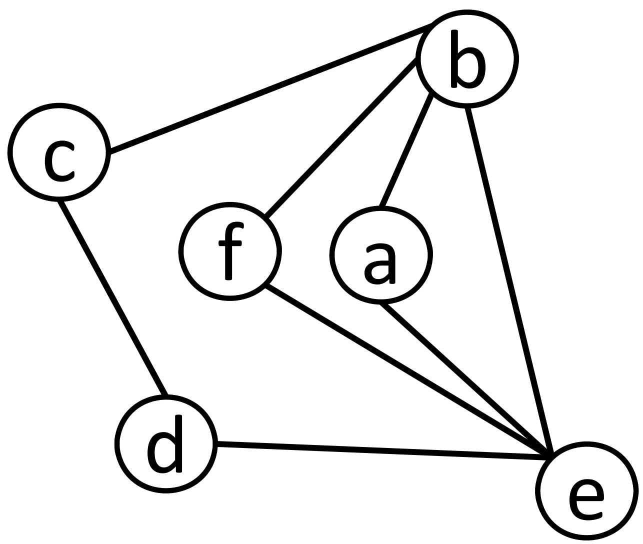
Image source: http://en.wikipedia.org/wiki/File:Konigsberg_bridges.png

Terminology of walks

walk: sequence of connected nodes/edges

closed walk: start and end point are the same

path: walk with no node used more than once



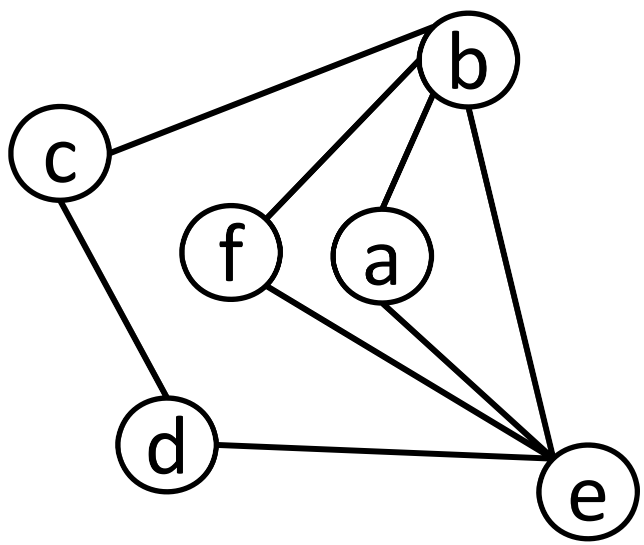
overhead

Terminology of walks

cycle: closed walk with 3+ nodes, no nodes except the first/last used more than once

acyclic: graph without cycles

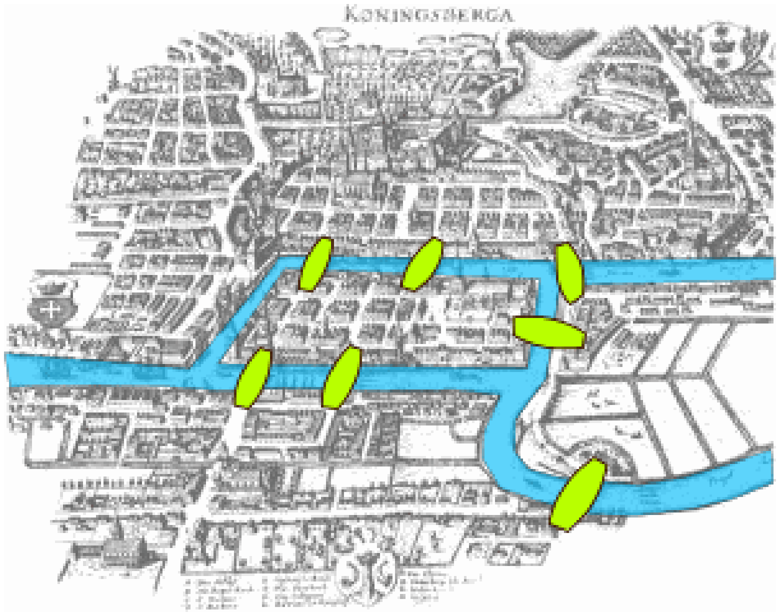
Euler circuit: closed walk that uses each *edge* exactly once



overhead

Bridges of Konigsberg

Possible to cross all bridges exactly once and end up where you started?



overhead

Properties of Euler Circuits

Let $G = (V, E)$ be a graph.

For G to have an Euler circuit, it is **necessary** that:

Properties of Euler Circuits

Let $G = (V, E)$ be a graph.

For G to have an Euler circuit, it is **necessary** that:

1. G is connected.

Properties of Euler Circuits

Let $G = (V, E)$ be a graph.

For G to have an Euler circuit, it is **necessary** that:

1. G is connected.
2. For all $v \in V$, $deg(v)$ is even.

Properties of Euler Circuits

Let $G = (V, E)$ be a graph.

For G to have an Euler circuit, it is **necessary** that:

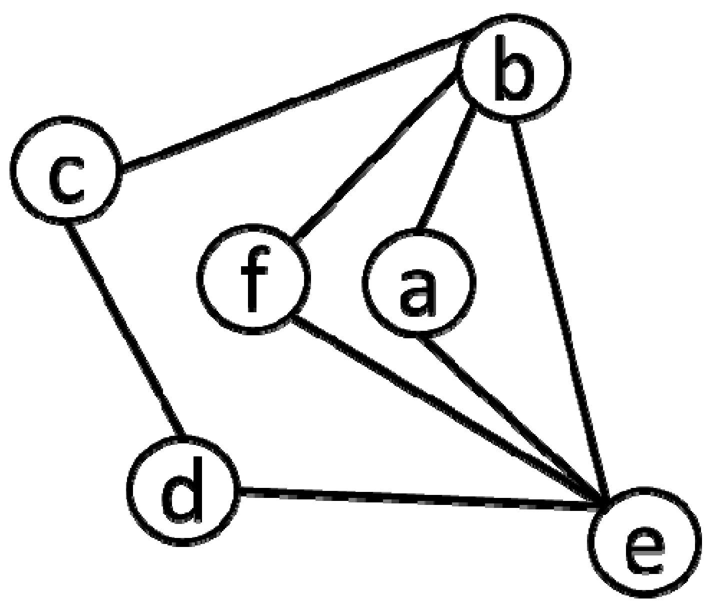
1. G is connected.
2. For all $v \in V$, $deg(v)$ is even.

   These conditions are also **sufficient**!

More connectivity terminology

The **distance** of two nodes is the minimum number of edges that forms a walk from one node to the other.

The **diameter** of a graph is the maximum distance between any two nodes in a graph.



More connectivity terminology

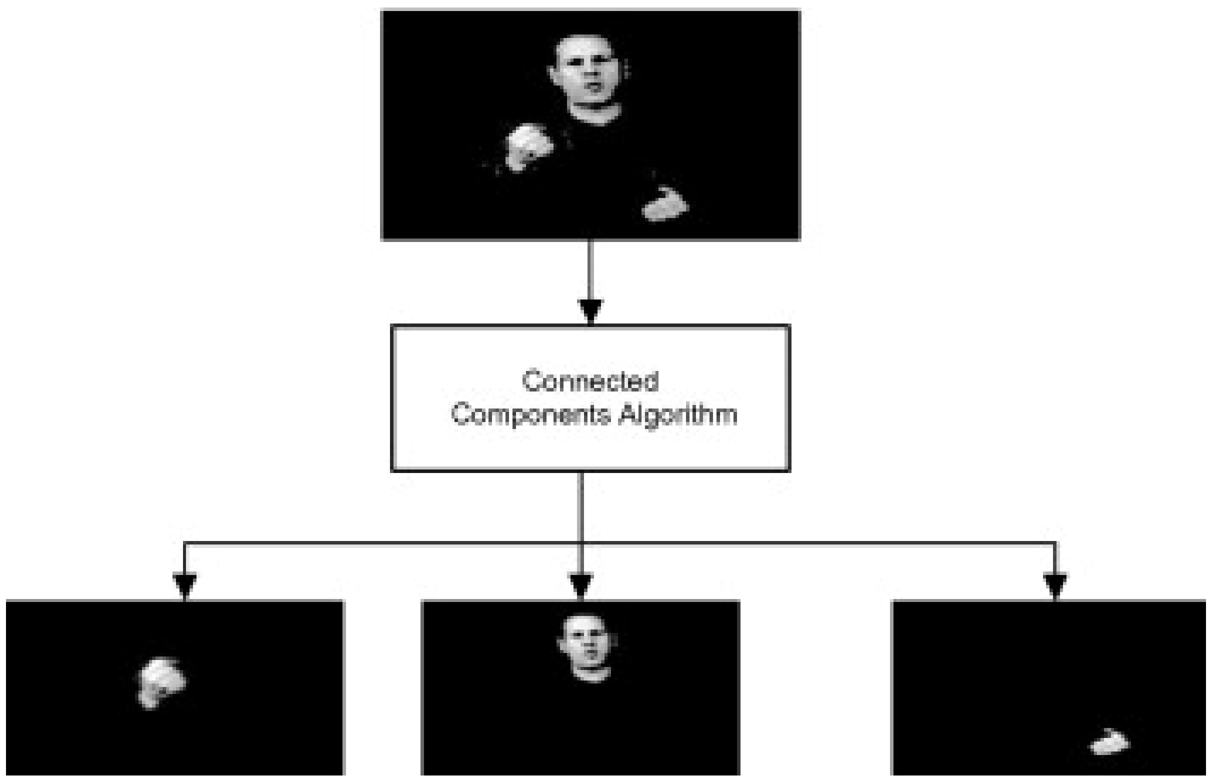
A graph is **connected** if there is a walk between any two nodes.

A **connected component** is a subset of nodes that are connected to each other but not connected to any other nodes.

overhead

Connected components application

- Many applications for finding connected pixels corresponding to boundaries or regions



Coloring

graph **coloring**: a labeling of nodes such that no two adjacent nodes have the same label

chromatic number: the minimum number of labels required for a coloring

cycles, wheels, complete, grid

overhead

Applications of coloring

- Making a map

- How many exam variations to make so that neighbors have different copies?

overhead

Bipartite graphs

$G = (V, E)$ is **bipartite** if we can split V into two non-overlapping sets V_1 and V_2, such that all edges connect $v_i \in V_1$ and $v_j \in V_2$

All graphs that can be two-colored are bipartite.

Complete bipartite graph $K_{m,n}$: $|V_1| = m$, $|V_2| = n$, all nodes in V_1 have edges to all nodes in V_2

overhead

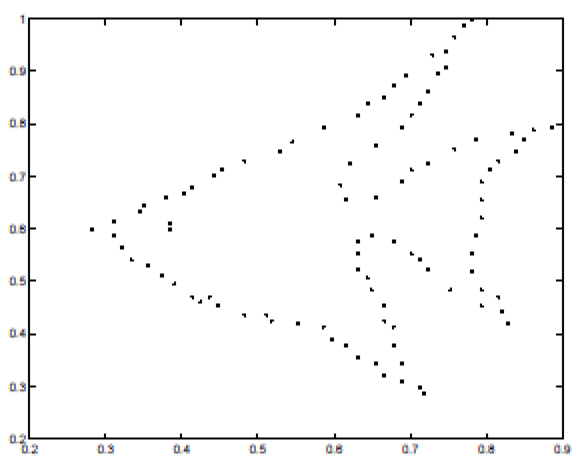
Application: bipartite graph matching

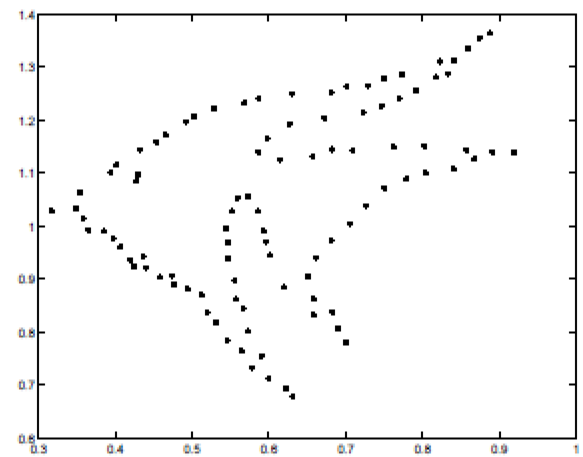
- The National Residency Matching Program
  - Med students give their preferences for residency jobs
  - Residency locations give their preferences for Med students
  - Solve for the maximum matching

overhead

Application: bipartite graph matching

- Computer vision
  - Get soft correspondence between points, with a cost of matching each point on left to each point on right
  - Find the “match” with the lowest cost





Cliques

Let $G = (V, E)$ be a graph.

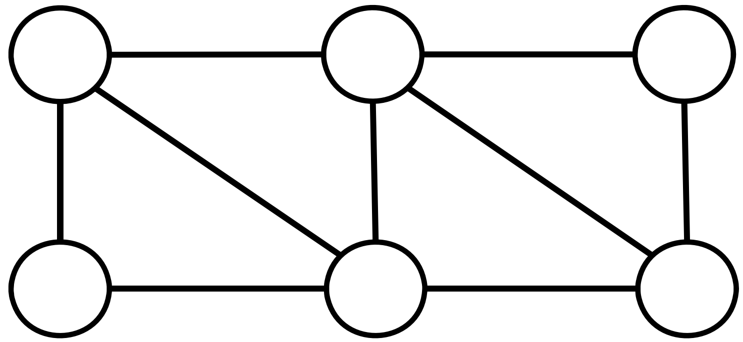
Clique a subset of nodes $C \subseteq V$ such that every two nodes in C are adjacent in G.

Equivalently: subgraph induced by C on G is **complete**.

Properties of Cliques

- used to model social groups (friends that all know each other)
- other applications in networking, chemistry, bioinformatics
- best known clique-finding algorithms run in **exponential** time
- hard to find even approximate solutions

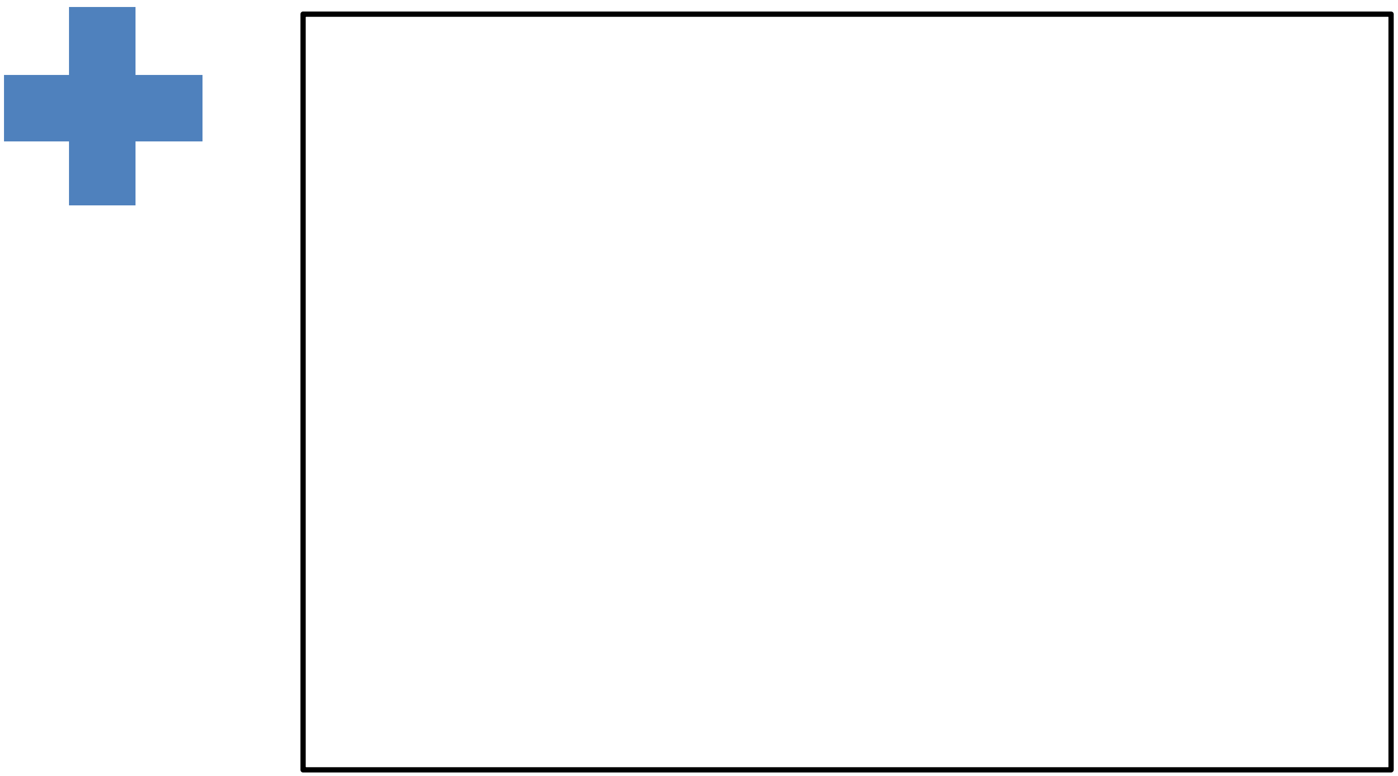
Two-way bounding: graph coloring



What is the chromatic number?

Two-way bounding: marker making

How many 3x3” crosses can be cut out of a 12x15” sheet of paper?



overhead

Two-way bounding: set equality

Claim: For any integer k, $\mathrm{A} = \{3ik + 5jk: i, j \text{ are integers}\}$ is equal to $\mathrm{B} = \{\text{multiples } of\ k\}$.

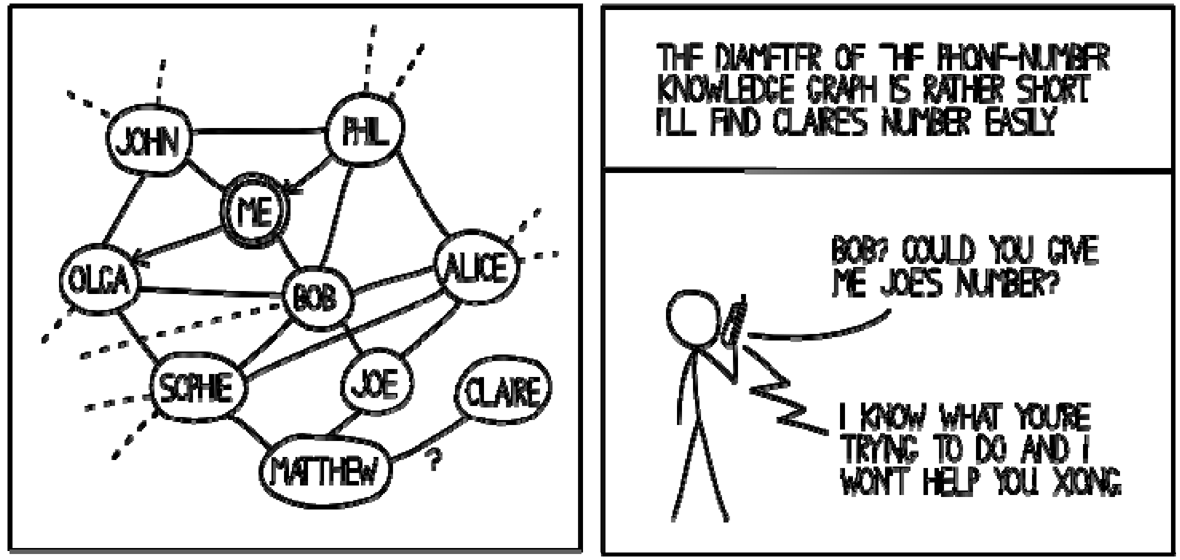
Things to remember

- Terminology for graph connectivity
  - Walk, path, cycle, acyclic, closed, Euler circuit, distance, diameter, connected components
- Graph coloring and how to apply it
- How to use two-way bounding in a variety of settings

Next class

- Induction

Short break



http://36758.comoj.com/20/

Chromatic Number?